Period 1, Feb 3, 2025

$$\int_{\frac{\pi}{2}}^{\pi} \cos x\,dx = \sin x + C \Big[_{\pi/2}^{\pi} = \sin\pi - \sin\frac{\pi}{2} = 0 - 1 = -1$$

$$\int_0^{\pi/2} \sin x\,dx = -\cos x + C \Big[_0^{\pi/2} = -\cos\frac{\pi}{2} - (-\cos 0) = -0 - (-1) = +1$$

$$\int_1^3 (2x^3 - 7x^2 + 2)\,dx = 2\cdot\frac{1}{4}\cdot x^{3+1=4} - 7\cdot\frac{1}{3}\cdot x^{2+1=3} + 2x + C \Big[_1^3$$

$$\frac{1}{2}x^4 - \frac{7}{3}x^3 + 2x + C \Big[_1^3$$

$$\frac{1}{2}(3)^4 - \frac{7}{3}(3)^3 + 2(3) - \left[\frac{1}{2}(1)^4 - \frac{7}{3}(1)^3 + 2(1)\right]$$

$$\frac{81}{2} - \frac{7}{3}(27) + 6 - \frac{1}{2} + \frac{7}{3} - 2 = \frac{81}{2} - \frac{1}{2} - 7\cdot 9 + 6 + \frac{7}{3} - 2$$

$$\frac{80}{2} - 63 + 4 + \frac{7}{3}$$

$$40 - 63 + 4 + \frac{7}{3}$$

$$-23 + 4 + \frac{7}{3}$$

$$-19 + 2\frac{1}{3} = -16\frac{2}{3} = -\frac{50}{3}$$

$$\int_1^4 (3x^4 - 2x + 1)\,dx$$

$$3\cdot\frac{1}{5}\cdot x^{4+1=5} - 2\cdot\frac{1}{2}\cdot x^{1+1=2} + x \Big[_1^4$$

$$\frac{3}{5}x^5 - x^2 + x \Big[_1^4$$

$$\frac{3}{5}(4)^5 - 4^2 + 4 - \left[\frac{3}{5}(1)^5 - 1^2 + 1\right]$$

$$\frac{3072}{5} - 16 + 4 - \frac{3}{5} \cancel{+1} \cancel{-1}$$

$$\frac{3069}{5} - 12 = \frac{3069}{5} - \frac{60}{5} = \frac{3009}{5} = 601\frac{4}{5}$$

$4\cdot 4 = 16$

$16\cdot 16 = 256$

$4^2\cdot 4^2 = 4^4$

$4\cdot 256 = 1024$

$1024\cdot 3 = 3072$

$$\int_3^6 \left(e^x + \frac{5}{x^2}\right)dx = \int_3^6 (e^x + 5x^{-2})dx = e^x + 5\cdot\frac{1}{-1}\cdot x^{-2+1=-1} + c$$

$$e^x - 5\cdot x^{-1} + c \Big[_3^6$$

$$e^6 - \frac{5}{6} - \left[e^3 - \frac{5}{3}\right] = e^6 - \frac{5}{6} - e^3 + \frac{5\cdot 2}{3\cdot 2} = e^6 - e^3 - \frac{5}{6} + \frac{10}{6}$$

$$e^6 - e^3 + \frac{5}{6}$$

$$\int_3^6 \left(e^x + \frac{2}{x^3}\right)dx = \int_3^6 (e^x + 2\cdot x^{-3})\,dx$$

$$e^x + \cancel{2}\frac{1}{\cancel{-2}}\cdot x^{-3+1=-2} + c \Big[_3^6 = e^x - x^{-2} + c \Big[_3^6 = e^x - \frac{1}{x^2} + c \Big[_3^6$$

$$e^6 - \frac{1}{6^2} - \left[e^3 - \frac{1}{3^2}\right] = e^6 - \frac{1}{36} - e^3 + \frac{1\cdot 4}{9\cdot 4}$$

$$e^6 - e^3 - \frac{1}{36} + \frac{4}{36} = e^6 - e^3 + \frac{3}{36} = e^6 - e^3 + \frac{1}{12}$$

7. If $h(x) = x^3$ has an average value of 12 on the interval $[0, k]$, then what is k?

$$\int_0^k (x^3)dx = \frac{1}{4}x^4 + c \Big[_0^k = \frac{1}{4}(k)^4 + c - \cancel{\left[\frac{1}{4}(0)^4 + c\right]}$$

$$48 = 8\cdot 6 = 2^3\cdot 6$$

$$\text{Average Value}\cdot(k-0) = \int_0^k x^3\,dx$$

$$4\cdot 12\cdot k = \frac{1}{\cancel{4}}\cdot k^4\cdot\cancel{4}$$

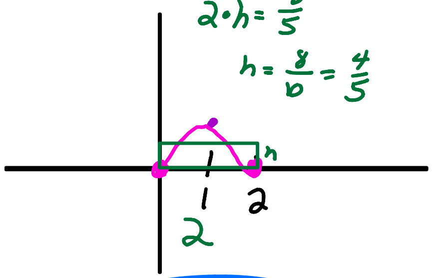
$$48k = k^4$$
$$-48k \quad -48k$$

$$0 = k^4 - 48k = k(k^3 - 48)$$

$$k = 0 \text{ or } k^3 - 48 = 0$$

$$k = \sqrt[3]{48} = \sqrt[3]{2^3\cdot 6} = 2\sqrt[3]{6}$$

6. What is the average value of the part of the graph of $g(x) = x^3(2-x)$ that lies in the first quadrant?



$2\cdot h = \frac{8}{5}$

$h = \frac{8}{10} = \frac{4}{5}$

$y = x^3(2-x)$

x	y
0	$0 = (0)(2)$
2	$0 = 2^3(2-2)$
1	$1 = 1^3(2-1)$

only zeros: 0, 2

$$\int_0^2 x^3(2-x)\,dx$$

$$\int_0^2 (2x^3 - x^4)\,dx$$

$$2\cdot\frac{1}{4}\cdot x^{3+1=4} - 1\cdot\frac{1}{5}\cdot x^{4+1} + C$$

$$\frac{1}{2}x^4 - \frac{1}{5}x^5 + C \Big]_0^2$$

$$\frac{1}{2}(2)^4 - \frac{1}{5}(2)^5 - \left[\cancel{\frac{1}{2}(0)^4 - \frac{1}{5}(0)^5}\right]$$

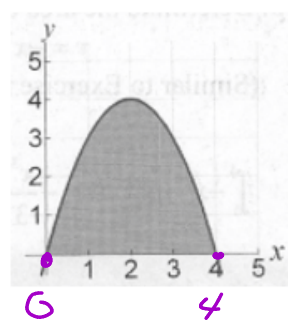
$$8 - \frac{32}{5} = 8 - 6\frac{2}{5} = 1\frac{3}{5}$$

$$\frac{40}{5} - \frac{32}{5} = \frac{8}{5} = 1\frac{3}{5}$$

2. Determine the area of the given region to the right:

$y = -x^2 + 4x$ $= x(-x+4)$

x	y
0	0
4	0



$$\int_0^4 (-x^2+4x)\,dx = -1\cdot\frac{1}{3}\cdot x^{2+1=3} + 4\cdot\frac{1}{2}\cdot x^{1+1=2} + C$$

$$-\frac{x^3}{3} + 2x^2 + C \Big]_0^4$$

$$-\frac{4^3}{3} + 2(4)^2 - \left[\cancel{-\frac{0^3}{3} + 2(0)^2}\right]$$

$$-\frac{64}{3} + 2(16) = -\frac{64}{3} + \frac{32\cdot 3}{1\cdot 3} = \frac{-64+96}{3} = \frac{32}{3} = 10\frac{2}{3}$$

4. Find the values of c guaranteed by the Mean Value Theorem for Integrals for the function over the given interval. $f(x) = \frac{1}{x^2}$, $[1,2]$.

$$\int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = 1\cdot\frac{1}{-1}\cdot x^{-2+1=-1} + c\Big[_1^2 = -x^{-1} + c = -\frac{1}{x} + c\Big[_1^2$$

$$-\frac{1}{2} - \left(-\frac{1}{1}\right) = -\frac{1}{2} + 1 = \frac{1}{2}$$

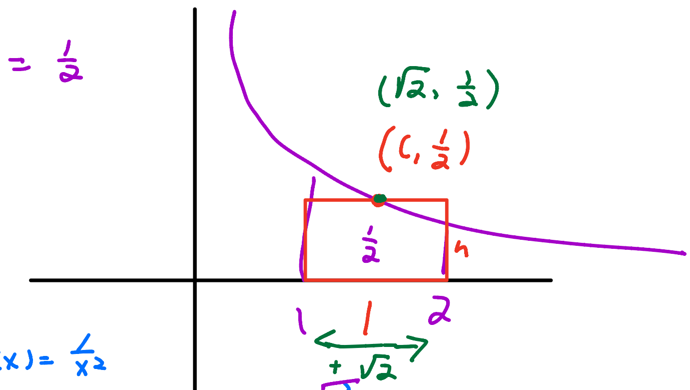
Average Value = height = h = $\frac{1}{2}$

$2-1=1$ = width

$h\cdot w = \frac{1}{2}$

$h\cdot 1 = \frac{1}{2}$

$h = \frac{1}{2}$



$F(x) = \frac{1}{x^2}$

$F(c) = \frac{1}{2} \Rightarrow \frac{1}{c^2} = \frac{1}{2} \Rightarrow \sqrt{c^2} = \sqrt{2} \Rightarrow c = \pm\sqrt{2}$

d) if $g(x) = \begin{cases} x+2, & -2 \le x < 2 \\ 2x, & 2 \le x < 11 \end{cases}$,

Then $\int_{-1}^{10} g(x)\,dx$ = ________

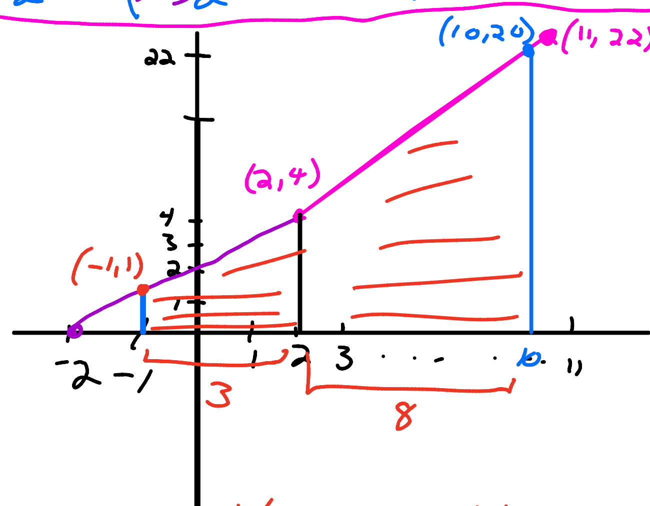
$$\int_{-1}^{2} (x+2)\,dx + \int_{2}^{10} 2x\,dx$$

$$\frac{1}{2}x^2 + 2x + c\Big[_{-1}^{2} + x^2 + c\Big[_2^{10}$$

$$\frac{1}{2}(2)^2 + 2(2) - \left[\frac{1}{2}(-1)^2 + 2(-1)\right] + 10^2 - 2^2$$

$$2 + \cancel{4} - \frac{1}{2} + 2 + 100 - \cancel{4}$$

$$104 - \frac{1}{2} = 103\frac{1}{2}$$



$\frac{1}{2}(1+4)3 + \frac{1}{2}(4+20)\cdot 8$

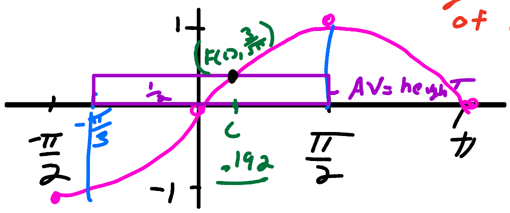
$\frac{5}{2}\cdot 3 + \frac{1}{2}(24)\cdot 8$

$\frac{15}{2} + 4(24)$

$7\frac{1}{2} + 96 = 103\frac{1}{2}$

5. What is the average value of $f(x) = \sin x$ on the interval $\left[-\frac{\pi}{3}, \frac{\pi}{2}\right]$?

height of rectangle



$$\int_{-\frac{\pi}{3}}^{\pi/2} \sin x\,dx = -\cos x + C \Big|_{-\frac{\pi}{3}}^{\pi/2}$$

$$-\cos\frac{\pi}{2} - \left(-\cos\left(-\frac{\pi}{3}\right)\right)$$

$$-0 + \frac{1}{2} = \frac{1}{2} = \text{Area}$$

$w = b - a$

$$\text{width} = \frac{\pi}{2} - \left(-\frac{\pi}{3}\right)$$

$$\frac{3\cdot\pi}{3\cdot 2} + \frac{\pi\cdot 2}{3\cdot 2} = \frac{5\pi}{6} = w$$

Area = Average value · width

$$\frac{6}{5\pi}\cdot\frac{1}{2} = h\cdot\cancel{\frac{5\pi}{6}}\cdot\cancel{\frac{6}{5\pi}}$$

$$\frac{3}{5\pi} = \frac{6}{10\pi} = h = \text{Average value}$$

$$\frac{1}{b-a}\cdot\int_a^b F(x)\,dx = \text{Average value}$$

$$\frac{1}{\frac{5\pi}{6}}\cdot\frac{1}{2} = \frac{\frac{1}{1}}{\frac{5\pi}{6}}\cdot\frac{1}{2} = \frac{1}{1}\cdot\frac{6}{5\pi}\cdot\frac{1}{2} = \frac{6}{10\pi} = \frac{3}{5\pi}$$

MVT

$$F(c) = \frac{3}{5\pi}$$

$$\sin c = \frac{3}{5\pi}$$

$$c = \sin^{-1}\frac{3}{5\pi}$$

$c = 0.192$

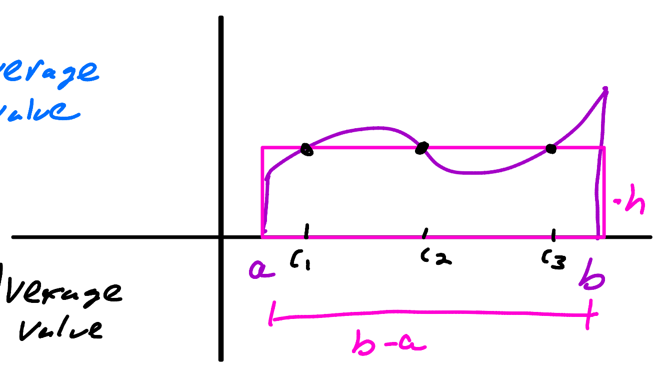
height = Average Value

MVT

$F(c_1)$ = Average Value

3 Times

Find c_1, c_2, c_3



$$\int_a^b F(x)\,dx = \text{Area under curve}$$

$$\frac{\cancel{(b-a)}\cdot h}{\cancel{(b-a)}} = \frac{\int_a^b F(x)\,dx}{(b-a)}$$

$$h = \frac{1}{b-a}\int_a^b F(x)\,dx$$

$$h = \text{Average Value} = \frac{1}{b-a}\int_a^b F(x)\,dx$$

Example 1: Liquid flows out of a tank at a rate of $40 - 2t$ gallons per minute, where $0 \le t \le 20$. Find the volume of liquid that flows out of the tank during the first 5 minutes.

$$\text{Rate} = 40 - 2T = \frac{dg}{dT}$$

$$\int_0^5 (40 - 2T)\,dT$$

$$40T - T^2 \Big|_0^5$$

$$40(5) - 5^2 - [\cancel{40(0)} - 0^2]$$

$$200 - 25 = 175 = \text{gallons}$$

$$\cancel{dT}\,(40 - 2T) = \frac{dg}{\cancel{dT}} \cdot \cancel{dT} \Rightarrow \text{gallons per minute}$$

$$\int (40 - 2T)\,dT = \int dg$$

$$\int (40 - 2T)\,dT = \text{gallons}$$

$\int_a^b v(T)\,dT = \text{displacement} \Rightarrow$ 6 steps Right, 4 steps Left

displacement = 2

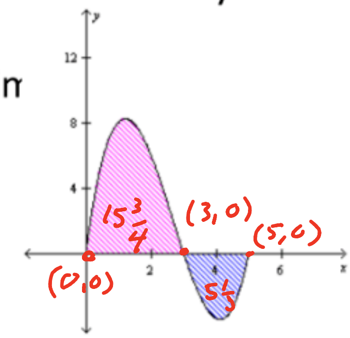
$\int_a^b |v(T)|\,dT = \text{distance} \Rightarrow$ 6 steps Right, 4 steps Left

distance = 10 steps

Example 3: A particle is moving along a line so that the velocity is $v(t) = t^3 - 8t^2 + 15t$ feet per second at time t.
What is the <u>total distance</u> of the particle on the tin
$0 \le t \le 5$? (without calculator or graph)

$$\int_1^5 |t^3 - 8t^2 + 15t|\, dt$$

$\frac{1}{4}T^4 - \frac{8}{3}T^3 + \frac{15}{2}T^2$



Make a number line to check if velocity function is above or below $x - axis$
Factor: $t^3 - 8t^2 + 15t = t(t-3)(t-5)$

$\int_0^3 (T^3 - 8T^2 + 15T)\,dT = \frac{1}{4}(3)^4 - \frac{8}{3}(3)^3 + \frac{15}{2}(3)^2$

$\frac{81}{4} - 72 + \frac{135}{2} = 15\frac{3}{4}$

$T^3 - 8T^2 + 15T$

$T(T^2 - 8T + 15)$

$T(T-5)(T-3) = 0$

$\int_3^5 (T^3 - 8T^2 + 15T)\,dT = \frac{1}{4}(5)^4 - \frac{8}{3}(5)^3 + \frac{15}{2}(5)^2 - \left[\frac{1}{4}(3)^4 - \frac{8}{3}(3)^3 - \frac{15}{2}(3)^2\right]$ $T = 0$ $T = 5$ $T = 3$

$\frac{625}{4} - \frac{1000}{3} + \frac{375}{2} - 15\frac{3}{4} = -5\frac{1}{3}$

From $1 \le t \le 5$, The particles total distance is $15\frac{3}{4} + 5\frac{1}{3} =$

av4-5-6-5_3_TheFundThmofCalc-usesurface.pptx?rlkey=xfqfwnjb0tzrnws4zqkd4uj1a&e=1&dl=0#

displacement $15\frac{3}{4} - 5\frac{1}{3} =$

Example 4.5: A particle is moving along a line so that the velocity is $v(t) = t^3 - 8t^2 + 15t$ feet per second at time t.
What is the average velocity from time $t = 0$ to $t = 5$

displacement $= 10\frac{5}{12} = \frac{125}{12}$

height · width
Average velocity · Time = displacement

Average velocity $\cdot \cancel{5} = \frac{125}{12 \cdot 5}$

Average velocity $= \frac{125}{60} = \frac{25}{12} = 2\frac{1}{12}$ FT/s